Ocean: Wind-driven circulation #2

ATM2106
Last time...

- **Ekman layer**: The layer near the surface that feels the wind stress
- **Ekman transport**: Wind drives ocean currents
  - Ocean transport to the right of the wind in NH.
  - Ocean transport to the left of the wind in SH.
- **Ekman pumping & suction**:
  - Convergence: Ekman pumping (downward)
  - Divergence: Ekman suction (upward)
What does Ekman pumping do to the interior of the ocean?
Remember that...

- Ekman pumping in the Ekman layer.
- Beneath the Ekman layer, the flow is in the geostrophic balance.

$w$ associated with Ekman pumping/suction

Geostrophic balance
Using momentum equations...

\[-fv = -\frac{1}{\rho_{ref}} \frac{\partial p}{\partial x}\]

\[fu = -\frac{1}{\rho_{ref}} \frac{\partial p}{\partial y}\]

\[\frac{\partial}{\partial y} (-fv) = -\frac{1}{\rho_{ref}} \frac{\partial}{\partial y} \left(\frac{\partial p}{\partial x}\right)\]

\[\frac{\partial}{\partial x} (fu) = -\frac{1}{\rho_{ref}} \frac{\partial}{\partial x} \left(\frac{\partial p}{\partial y}\right)\]

\[\frac{\partial}{\partial y} (-fv) = \frac{\partial}{\partial x} (fu)\]
Using momentum equations...

\[
\frac{\partial}{\partial y}(-fv) = \frac{\partial}{\partial x}(fu)
\]

\[
-f \frac{\partial v}{\partial y} - \beta v = f \frac{\partial u}{\partial x}
\]

\[
-f \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = \beta v
\]

\[
f \frac{\partial w}{\partial z} = \beta v
\]

If we assume that

\[
f = f_0 + \beta y
\]

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0
\]
1. Ekman pumping \((w_{\text{EK}} < 0)\)

\[
f \frac{\partial w}{\partial z} = \beta v
\]

Ekman pumping leads to \(v < 0\) in the NH \((f > 0)\).

\[
v = \frac{f}{\beta} \frac{w_{\text{EK}}}{H}
\]

\[
w = w_{\text{EK}}
\]

\[
w = 0
\]
1. Ekman suction \((w_{EK} > 0)\)

\[
f \frac{\partial w}{\partial z} = \beta v
\]

Ekman pumping leads to \(v > 0\) in the NH \((f > 0)\).

\[
v = \frac{f \, w_{EK}}{\beta \, H}
\]
Ekman pumping / suction

![Map showing Ekman suction](image-url)
How fast is $v$ driven by Ekman pumping/suction?

$v = \frac{f}{\beta} \frac{w_{EK}}{H}$

1 $\times$ 10^{-4} s^{-1} \quad 30 \text{ m yr}^{-1}

2 $\times$ 10^{-11} m^{-1} s^{-1} \quad 1000 \text{ m}

$v = O(1 cm/s)$

(Thermocline depth)
The depth-integrated circulation

- To simplify the ocean circulation, let’s consider the vertical integration of the momentum equation.

\[-fv + \frac{1}{\rho_{\text{ref}}} \frac{\partial p}{\partial x} = \frac{1}{\rho_{\text{ref}}} \frac{\partial \tau_x}{\partial z}\]

\[f u + \frac{1}{\rho_{\text{ref}}} \frac{\partial p}{\partial y} = \frac{1}{\rho_{\text{ref}}} \frac{\partial \tau_y}{\partial z}\]

\[-f \frac{\partial v}{\partial y} - \beta v + \frac{1}{\rho_{\text{ref}}} \frac{\partial^2 p}{\partial x \partial y} = \frac{1}{\rho_{\text{ref}}} \frac{\partial}{\partial z} \left( \frac{\partial \tau_x}{\partial y} \right)\]

\[f \frac{\partial u}{\partial x} + \frac{1}{\rho_{\text{ref}}} \frac{\partial^2 p}{\partial x \partial y} = \frac{1}{\rho_{\text{ref}}} \frac{\partial}{\partial z} \left( \frac{\partial \tau_y}{\partial x} \right)\]

\[\beta v = f \frac{\partial w}{\partial z} + \frac{1}{\rho_{\text{ref}}} \frac{\partial}{\partial z} \left( \frac{\partial \tau_y}{\partial x} - \frac{\partial \tau_x}{\partial y} \right)\]
The depth-integrated circulation

\[ \beta v = f \frac{\partial w}{\partial z} + \frac{1}{\rho_{\text{ref}}} \frac{\partial}{\partial z} \left( \frac{\partial \tau_y}{\partial x} - \frac{\partial \tau_x}{\partial y} \right) \]

Vertical integration

\[ \int_{-D}^{0} \beta v dz = \int_{-D}^{0} f \frac{\partial w}{\partial z} dz + \int_{-D}^{0} \frac{1}{\rho_{\text{ref}}} \frac{\partial}{\partial z} \left( \frac{\partial \tau_y}{\partial x} - \frac{\partial \tau_x}{\partial y} \right) dz \]

\[ \beta V = \frac{1}{\rho_{\text{ref}}} \left( \frac{\partial \tau_{\text{wind},y}}{\partial x} - \frac{\partial \tau_{\text{wind},x}}{\partial y} \right) \]

\[ w(z=0) = w(z=-D) = 0 \]
\[ \tau(z = -D) = 0 \]
Sverdrup relation

\[ \beta V = \frac{1}{\rho_{ref}} \left( \frac{\partial \tau_{wind,y}}{\partial x} - \frac{\partial \tau_{wind,x}}{\partial y} \right) \]

The depth-integrated meridional transport is determined by wind stress when we assume

1. Coriolis force, pressure gradient force and stress are balanced.
2. Homogeneous fluid (so density is constant)
3. No vertical flow at the surface and the bottom
4. Stress is zero at the bottom
Sverdrup relation

\[ \beta V = \frac{1}{\rho_{ref}} \left( \frac{\partial \tau_{\text{wind},y}}{\partial x} - \frac{\partial \tau_{\text{wind},x}}{\partial y} \right) \]

- Polar Easterlies
- Westerlies (Roaring Forties)
- Trade Winds
- Doldrums

Ekman transport

Wind stress curl

Upwelling

Downwelling
Wind stress curl, geostrophic current, Sverdrup relation

\[ \frac{\partial \tau_y}{\partial x} - \frac{\partial \tau_x}{\partial y} < 0 \]

\[ \frac{\partial \tau_y}{\partial x} - \frac{\partial \tau_x}{\partial y} > 0 \]

- Northern H.: equatorward
- Southern H.: poleward

- Northern H.: poleward
- Southern H.: equatorward
Sverdrup relation

\[ \beta V = \frac{1}{\rho_{ref}} \left( \frac{\partial \tau_{wind,y}}{\partial x} - \frac{\partial \tau_{wind,x}}{\partial y} \right) \]

wind stress curl

Prevailing Zonal Winds

- Polar Easterlies
- Westerlies (Roaring Forties)
- Trade Winds
- Doldrums

Ekman transport

upwelling
downwelling

No meridional transport

(+)

(-)

(+)

(-)
Sverdrup relation

\[ \beta V = \frac{1}{\rho_{\text{ref}}} \left( \frac{\partial \tau_{\text{wind},y}}{\partial x} - \frac{\partial \tau_{\text{wind},x}}{\partial y} \right) \]
Why does it have to be on the west side?
Green: eastward  
brown: westward  

Green: northward  
brown: southward
Schematic of surface circulation (modified from Schmitz, 1995)
Stratified ocean and eddies
Homogeneous ocean v.s. ocean with eddies
Baroclinic instability

- Stratified ocean has tilted isopycnal (same density line).
- It means that there is **available potential energy (APE)**.
Available potential energy

- It is responsible for the atmospheric eddies.
- It is also responsible for the oceanic eddies.
- The stratified fluid with tilted isopycnal has higher potential energy than that with flat isopycnal.
A schematic of the mechanism maintaining the tilt isopycnal
# Atmospheric eddies v.s. oceanic eddies

<table>
<thead>
<tr>
<th></th>
<th>Atmospheric eddies</th>
<th>Oceanic eddies</th>
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</thead>
<tbody>
<tr>
<td>Length scale</td>
<td>700 km</td>
<td>50 km</td>
</tr>
<tr>
<td>Time scale</td>
<td>1 day</td>
<td>A week</td>
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<tr>
<td>Source</td>
<td>APE by radiative imbalance</td>
<td>Mechanically produced APE</td>
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<tr>
<td>Role</td>
<td>Heat and momentum transport</td>
<td>Heat and momentum transport but less crucial than those in the atmosphere</td>
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